

Equilibrium in x direction

Equilibrium in y direction

and

$$dy = dy' \cos(\theta), \quad dx = dx' \cos(\theta)$$

Give the rotation relations in the next page

ROTATION FROM x-y TO x'-y'

$$\sigma'_{xx} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta)$$

$$\sigma'_{yy} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos(2\theta) - \tau_{xy} \sin(2\theta)$$

$$\tau'_{yy} = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

PRINCIPAL STRESS SYSTEM

$$\tau'_{yy} = 0 = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

$$\tan(2\theta) = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

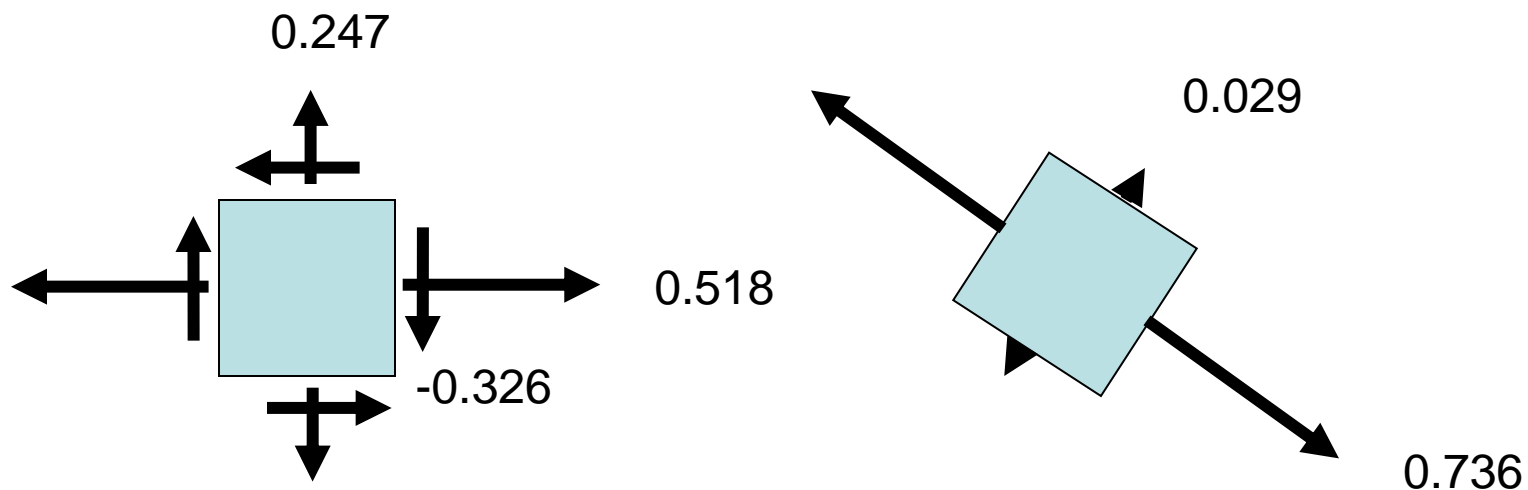
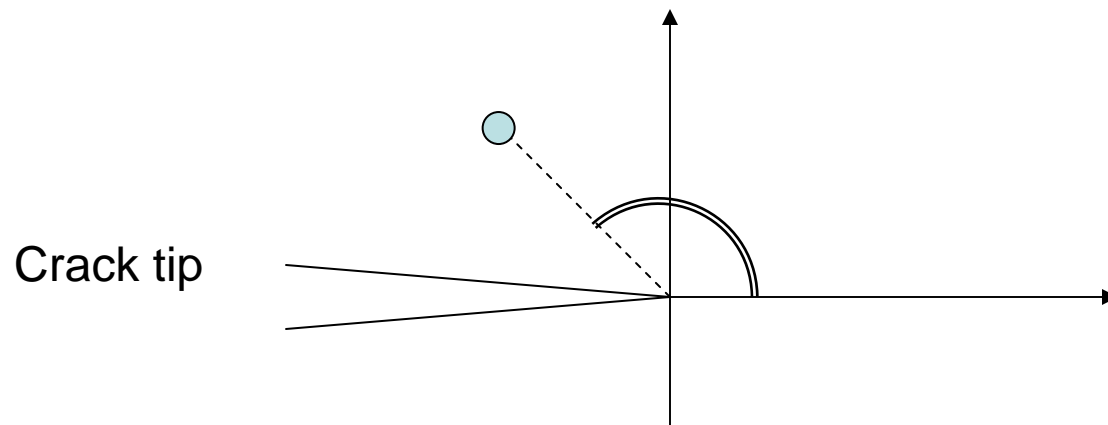
Direction of
max principal
stress

$$\theta = \frac{1}{2} \arctan\left(\frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}\right)$$

Principal stresses

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$



To connect with the math of the previous course

Principal stresses are the eigenvalues of the following system

$$\begin{bmatrix} (\sigma_{xx} - \sigma) & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & (\sigma_{yy} - \sigma) & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & (\sigma_{zz} - \sigma) \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0$$

Principal stresses (eigenvalues) are obtained by solving the following equation

$$\det \begin{bmatrix} (\sigma_{xx} - \sigma) & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & (\sigma_{yy} - \sigma) & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & (\sigma_{zz} - \sigma) \end{bmatrix} = 0$$

To connect with the math of the previous course
(for 2D problems, e.g. plane strain)

$$\det \begin{bmatrix} (\sigma_{xx} - \sigma) & \sigma_{xy} & 0 \\ \sigma_{xy} & (\sigma_{yy} - \sigma) & 0 \\ 0 & 0 & (\sigma_{zz} - \sigma) \end{bmatrix} = 0 \Leftrightarrow ((\sigma_{xx} - \sigma)(\sigma_{yy} - \sigma) - \sigma_{xy}^2)(\sigma_{zz} - \sigma) = 0$$

i.e., principal stresses are
given by:

$$\sigma_3 = \sigma_{zz} \quad \text{and} \\ \sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

If all this fails to register please visit :

http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/calc_principal_stress.cfm

